



$$K_L \rightarrow \mu^\pm e^\mp \nu \bar{\nu} \text{ as background to } K_L \rightarrow \mu^\pm e^\mp$$

S. Dawson^{(a)*} and G. Valencia^(b)

^(a) *Physics Department*

Brookhaven National Laboratory

Upton, NY 11973

^(b) *Theoretical Physics*

Fermi National Accelerator Laboratory

Batavia, IL 60510

Abstract

We consider the process $K_L \rightarrow \mu^\pm e^\mp \nu \bar{\nu}$ at next to leading order in chiral perturbation theory. This process occurs in the standard model at second order in the weak interaction and constitutes a potential background in searches for new physics through the modes $K_L \rightarrow \mu^\pm e^\mp$. We find that the same cut, $M_{\mu e} > 489$ MeV, used to remove the sequential decays $K_{l3} \rightarrow \pi_{l2}$ pushes the $B(K_L \rightarrow \mu^\pm e^\mp \nu \bar{\nu})$ to the 10^{-23} level, effectively removing it as a background.

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1 Introduction

In the minimal standard model with massless neutrinos, the lepton family number is absolutely conserved so the decays $K_L \rightarrow \mu^\pm e^\mp$ do not occur. Their observation would constitute clear evidence for new physics. In view of this, there have been several experiments studying these modes and others are planned for the future. The current experimental upper bounds are:

$$B(K_L \rightarrow \mu^\pm e^\mp) < \begin{cases} 9.7 \times 10^{-11} & \text{KEK-137 [1]} \\ 3.3 \times 10^{-11} & \text{AGS-791 [2]} \end{cases} \quad (1.1)$$

This latter number represents the most sensitive kaon experiment to date. The proposed successor experiment, AGS-871, expects to improve the sensitivity by a factor of about 20.

A model independent study of this type of processes can be done following the approach of Buchmüller and Wyler, Ref. [3]. The physics beyond the standard model is parameterized by an effective Lagrangian that is gauge invariant under $SU_c(3) \times SU_L(2) \times U_Y(1)$. This Lagrangian is given by a sum of four fermion operators, of which we present the purely left handed one as an example:

$$\mathcal{O}_{V-A} = C_{V-A} \frac{g^2}{\Lambda^2} \bar{\mu} \gamma_\mu \frac{(1 + \gamma_5)}{2} e \bar{s} \gamma^\mu \frac{(1 + \gamma_5)}{2} d \quad (1.2)$$

The factor g^2 is included to reflect the fact that we think of these operators as originating in the exchange of a heavy gauge boson (or perhaps a scalar) in the new physics sector. Any additional factors, like mixing angles, are contained in the coefficient C_i . It is conventional to assume that C_i is of order $\mathcal{O}(1)$, so that Λ is the scale that characterizes the heavy degrees of freedom (typically the mass of the exchanged boson). One then interprets the bounds on the decays induced by these operators as bounds on the “scale of new physics” Λ .

It is standard to compare this mode to the rate for $K^+ \rightarrow \mu^+ \nu$, to absorb the hadronic matrix element as well as common kinematical factors in the limit $m_e = 0$.

One finds:

$$\frac{\Gamma(K_L \rightarrow \mu^+ e^-)}{\Gamma(K^+ \rightarrow \mu^+ \nu)} = 2 \frac{C_{V-A}^2}{|V_{us}|^2} \left(\frac{m_W}{\Lambda} \right)^4 \quad (1.3)$$

The current experimental limit Eq. 1.1 then implies the bound $\Lambda > 108 \text{ TeV}$. This result can be interpreted as a bound on the mass of new particles in different models [4].

These decays are also allowed in minimal extensions of the Standard Model in which the neutrinos are given a mass. The decays then proceed via one-loop box diagrams. The decay rate for $K_L \rightarrow \mu^\pm e^\mp$ is proportional to the product of mixing angles between the μ , e and the heavy neutral lepton N : $|U_{Ne} U_{N\mu}^*|^2$. Using the result $B(\mu \rightarrow e \gamma) < 4.9 \times 10^{-11}$ [5], the authors of Ref. [6] find $|U_{Ne} U_{N\mu}^*|^2 < 7 \times 10^{-6}$ for $m_N > 45 \text{ GeV}$. From this they conclude that $B(K_L \rightarrow \mu^\pm e^\mp)$ is at most 10^{-15} in this type of models. Marciano [7] has pointed out that there is a better bound $|U_{Ne} U_{N\mu}^*|^2 < 10^{-8}$ coming from $\mu \rightarrow e$ conversion in the field of a heavy nucleus. With this bound one finds $B(K_L \rightarrow \mu^\pm e^\mp)$ to be at most a few times 10^{-18} . In left-right symmetric models $B(K_L \rightarrow \mu^\pm e^\mp)$ can be as large as 10^{-13} , although this happens only in a small corner of parameter space [8].

Given the level of sensitivity expected for future experiments, as well as the small rates predicted in many models, it is important to study processes that could fake this one at the 10^{-15} level. The ultimate background for the decays $K_L \rightarrow \mu^\pm e^\mp$ is the standard model process $K_L \rightarrow \mu^\pm e^\mp \nu_\mu \nu_e$, which we study in this paper using the techniques of chiral perturbation theory (χ PT).

2 Chiral Lagrangian

The lowest order chiral Lagrangian, $\mathcal{O}(p^2)$, is [9, 10]:

$$\mathcal{L}_S^{(2)} = \frac{f_\pi^2}{4} \text{Tr} \left(D_\mu U D^\mu U^\dagger \right) + B_0 \frac{f_\pi^2}{2} \text{Tr} \left(MU + U^\dagger M \right). \quad (2.1)$$

M is the diagonal matrix (m_u, m_d, m_s) , and the meson fields are contained in the matrix $U = \exp(2i\phi/f_\pi)$ with:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & 2\eta/\sqrt{6} \end{pmatrix}. \quad (2.2)$$

U transforms under the chiral group as $U \rightarrow RUL^\dagger$. For our purpose, the charged W^\pm bosons will be external fields, contained in the covariant derivative:

$$D_\mu U = \partial_\mu U + iU l_\mu$$

$$l_\mu = -\frac{e}{\sqrt{2} \sin \theta_W} \left(W_\mu^+ T + W_\mu^- T^\dagger \right), \quad (2.3)$$

where T is the matrix:

$$T = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (2.4)$$

For the pion decay constant we use $f_\pi = 93$ MeV.

At next to leading order, $\mathcal{O}(p^4)$, there are ten more operators [10] in the normal intrinsic parity sector. For the process we consider only one out of the ten terms contributes:

$$\mathcal{L}_S^{(4)} = -iL_9 \text{Tr} \left(F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \right) \quad (2.5)$$

Since there are no photons involved, the field strength tensor $F_{\mu\nu}$ is given by:

$$F_L^{\mu\nu} = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu] \quad (2.6)$$

At this same order there is also a contribution from the anomaly. The contribution from the anomaly is given by the Wess-Zumino-Witten anomalous action [11]. It contributes the following term to the process we study:

$$\mathcal{L}_{WZW}^{(4)} = -\frac{g^2 N_c}{48\pi^2 f_\pi} \text{Re}(V_{ud}V_{us}^*) \epsilon^{\mu\nu\alpha\beta} \left(\partial_\mu W_\nu^- W_\alpha^+ + W_\mu^- \partial_\nu W_\alpha^+ \right) \partial_\beta K_2^0, \quad (2.7)$$

where K_2^0 is the CP odd neutral kaon (we ignore CP violation) and $N_c = 3$.

A complete calculation to $\mathcal{O}(p^4)$ consists of tree-level diagrams with vertices from Eqs. 2.1, 2.5, and 2.7, and of one-loop diagrams using only Eq. 2.1. For the purpose of our paper it will be sufficient to ignore the one-loop contributions. In order for this to be consistent we use values for L_9 that are scale independent and derived from tree-level models. In particular, we will use $L_9 = 7.3 \times 10^{-3}$ from vector meson resonance saturation models and $L_9 = 6.3 \times 10^{-3}$ from quark models.

3 $K_L \rightarrow \mu^\pm e^\mp \nu \bar{\nu}$

To leading order in χ PT this process is dominated by $K_L \rightarrow \pi^\pm e^\mp \nu_e$ followed by $\pi^\pm \rightarrow \mu^\pm \nu_\mu$. The branching ratio for this chain can be estimated using the narrow width approximation to be a huge 38% (we have summed over the two modes). The processes with $\mu \leftrightarrow e$ exchanged are helicity suppressed and are typically ignored. It is easy to see that the maximum invariant mass of the lepton pair in this sequential decay is $m_{\mu e} < 489 \text{ MeV}$. It is therefore possible to remove this background with a cut on the lepton pair invariant mass. Going beyond the narrow width approximation,

and including next to leading order terms in χ PT can yield a lepton pair invariant mass larger than 489 MeV.

In terms of the momenta of the particles involved, $K_L(k) \rightarrow e^-(p_e)\bar{\nu}_e(q_e)\mu^+(p_m)\nu_m(q_m)$ it is convenient to define $p^+ = q^m + p^m$, $p^- = q^e + p^e$, and to use the variables:

$$y_m = \frac{(p^+)^2}{m_K^2}, \quad y_e = \frac{(p^-)^2}{m_K^2}, \quad (3.1)$$

$$\nu = \frac{2k \cdot (q_e - p_e)}{m_K^2}. \quad (3.2)$$

We normalize all masses to the K_L mass defining for each particle $P = e, \mu, \pi^+, K^+$, $r_P \equiv m_P/m_{K_L}$. We also use the leptonic currents:

$$\begin{aligned} L_\mu^e &= \bar{u}_e \gamma_\mu (1 - \gamma_5) v_\nu \\ L_\mu^m &= \bar{u}_\nu \gamma_\mu (1 - \gamma_5) v_m \\ \hat{L}^m &= \bar{u}_\nu (1 + \gamma_5) v_m \end{aligned} \quad (3.3)$$

The lowest order matrix element is given by the sum of a charged pion and a charged kaon poles as in Fig. 1a:

$$M^{(2)} = G_F^2 s_\theta c_\theta \left(\frac{f_\pi}{y_m - r_\pi^2 + i\Gamma_\pi r_\pi} + \frac{f_K}{y_m - r_{K^+}^2 + i\Gamma_{K^+} r_{K^+}} \right) \frac{m_\mu}{m_K^2} (p^+ \cdot L^e) \hat{L}^m \quad (3.4)$$

where s_θ is the sine of the Cabibbo angle, $s_\theta \approx .22$. The explicit factor m_μ reflects the helicity suppression that allows us to drop the diagrams with $\mu - e$ interchanged. Neglecting the electron mass, we then find:

$$\Gamma = \frac{G_F^4 m_\mu^2 m_{K_L}^5 f_\pi^2}{1536\pi^5} |s_\theta c_\theta|^2 I \quad (3.5)$$

with

$$I = \int_{r_\mu^2}^1 \frac{dy}{y} (y - r_\mu^2)^2 \left[-3y^2 \log y + \frac{1}{4}(1 - y^2)(1 - 8y + y^2) \right]$$

$$\left| \frac{1}{y - r_\pi^2 + ir_\pi \frac{\Gamma_{\pi^+}}{m_{K_L}}} + \frac{(f_K/f_\pi)}{y - 1 + ir_{K^+} \frac{\Gamma_{K^+}}{m_{K_L}}} \right|^2 \quad (3.6)$$

The last integral can be performed analytically if we use the narrow width approximation. Doing so we find a $B(K_L \rightarrow \mu^+ e^- \nu \bar{\nu})$ of 18% from the π^+ pole, and 1.8×10^{-9} from the K^+ pole. However, as noted before, the largest $\mu - e$ invariant mass from these contributions is $M_{\mu e}^2(max) = m_K^2 + m_\mu^2 - m_p^2$. Therefore $M_{\mu e} < 489$ MeV for the pion pole and $M_{\mu e} < 123$ MeV for the K^+ pole.

We can estimate the size of the off-shell contributions from the lowest order matrix element, by implementing a “theorist’s cut” of ± 1 MeV around the pole mass. We then obtain $B(K_L \rightarrow \mu^+ e^- \nu \bar{\nu}) = 1.62 \times 10^{-15}$. However, when we use the realistic cut $M_{\mu e} > 489$ MeV instead, this number is dramatically reduced to $B(K_L \rightarrow \mu^+ e^- \nu \bar{\nu}) = 8.7 \times 10^{-24}$.

At next to leading order in χ PT, we obtain the first structure dependent contributions depicted schematically in Fig. 1b. We find:

$$M^{(4)} = \frac{G_F^2}{f_\pi} s_\theta c_\theta \left\{ 4L_9 \left[m_K^2 (y_e - y_m) (L^e \cdot L^m) - m_\mu (p^+ \cdot L^e) \hat{L}^m \right] - \frac{i}{2\pi^2} \epsilon^{\mu\beta\rho\sigma} p_\mu^+ p_\beta^- L_\rho^m L_\sigma^e \right\} \quad (3.7)$$

where the first term comes from Eq. 2.5, and the second term comes from the anomaly Eq. 2.7.

Beyond leading order, the rate receives contributions from Eq. 3.7 and from its interference with the lowest order matrix element. With no cuts, we find numerically (omitting the lowest order p^2 contribution):

$$B^{(4)}(K_L \rightarrow \mu^+ e^- \nu \bar{\nu}) \approx 5.1 \times 10^{-15} L_9 + 8.4 \times 10^{-14} L_9^2 + 4.5 \times 10^{-18} \quad (3.8)$$

With a cut $M_{\mu e} > 489$ MeV, and taking $L_9 = 0.007$ we find for the complete rate to $\mathcal{O}(p^4)$ $B(K_L \rightarrow \mu^+ e^- \nu \bar{\nu}) \approx 9 \times 10^{-24}$.

Although the standard model process $K_L \rightarrow \mu^+ e^- \nu \bar{\nu}$ is a potential background for the lepton flavor number violating decay $K_L \rightarrow \mu^+ e^-$, we have found that the cut in the invariant mass of the lepton pair $M_{\mu e} > 489$ MeV, necessary to remove the sequential decays $K_{\ell 3} \rightarrow \pi_{\ell 2}$ background, also suppresses this decay well below expected sensitivities.

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Figure Captions

- Fig. 1: Diagrams contributing to $K_L \rightarrow \mu^\pm e^\mp \nu \bar{\nu}$. a) Pole diagrams at order $\mathcal{O}(p^2)$.
b) Direct, or structure dependent contributions at order $\mathcal{O}(p^4)$.

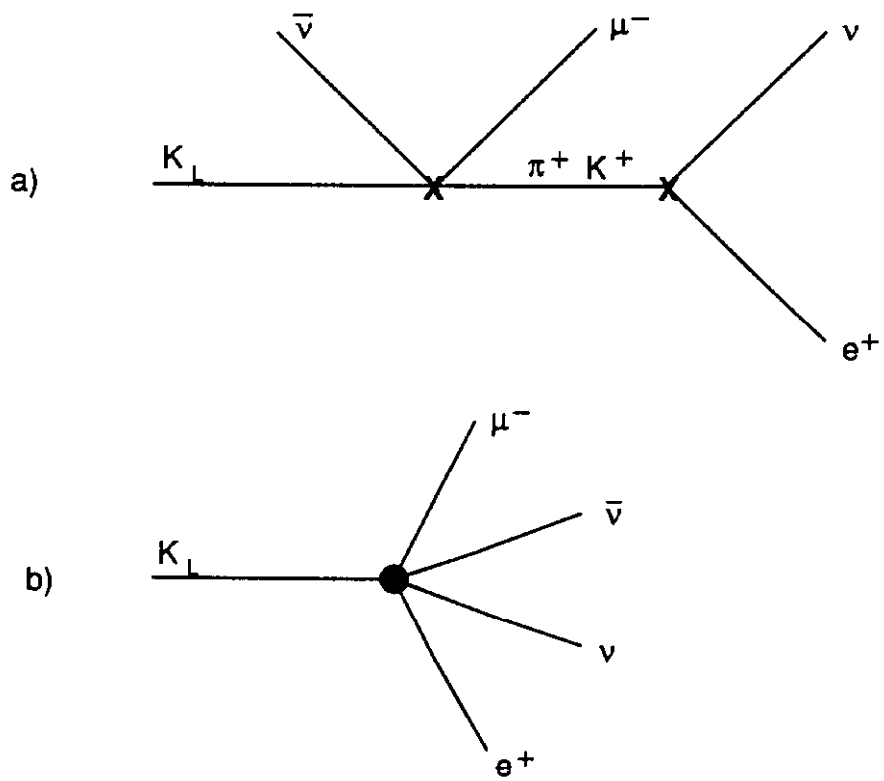


Figure 1.